INTERNATIONAL CHINESE STATISTICAL ASSOCIATION 泛華統計協會 2024年国际泛华统计协会（ICSA）中国会议

A Bayesian estimation approach of random switching exponential smoothing with application to credit forecast

Renhe Wang [a], Tong Wang [b], Zhiyong Qian [a], Shulan Hu [a]

[a] Zhongnan University of Economics and Law, Wuhan, 430073, Hubei, People's Republic of China

[b] Business School, University of Edinburgh, Edinburgh, EH8 9JS, Scotland, United Kingdom

Introduction

Exponential smoothing techniques are pivotal in forecasting within economic, financial, and operational management domains. The evolution from Holt's initial model to the Single Source of Error (SSOE) and subsequently the Multiple Source of Error (MSOE) frameworks reflects significant advancements in handling the dynamic aspects of time series data. Our research focuses on the MSOE model, specifically its application through **Random Coefficient Markov Chain Monte Carlo (RC-MCMC) methods**. This method leverages banded precision matrices to enhance the estimation efficiency of model parameters. Our simulations, alongside empirical applications using **quarterly credit-to-GDP data from the Bank for International Settlements**, demonstrate the RC-MCMC's superior accuracy in parameter estimation compared to the direct RC-SSPACE method. This study underscores the RC-MCMC's practical relevance and robustness in economic time series analysis.

Bayesian estimation

Now we discuss an effective posterior sampler in model (1). Such estimation method was first introduced in the stochastic volatility state-space model by Kim et al. (1998), and have been further improved by algorithmic advances in Chan and Jeliazkov (2009) and Chan and Grant (2016).

We develop a MCMC algorithm in which posterior draws can be obtained by sequentially sampling from:

1. Step 1: $P(\phi|\boldsymbol{A})$;
2. Step 2: $P(\boldsymbol{A}|\boldsymbol{b},\phi,\sigma_\xi^2)$;
3. Step 3: $P(\boldsymbol{l}|\boldsymbol{y},\boldsymbol{A},\sigma_\epsilon^2,\sigma_\eta^2)$;
4. Step 4: $P(\boldsymbol{b}|\boldsymbol{l},\boldsymbol{A},\sigma_\eta^2,\sigma_\xi^2)$;
5. Step 5: $P(\sigma_\epsilon^2|\boldsymbol{y},\boldsymbol{l},\boldsymbol{b},\boldsymbol{A})$;
6. Step 6: $P(\sigma_\eta^2|\boldsymbol{l},\boldsymbol{b},\boldsymbol{A})$;
7. Step 7: $P(\sigma_\xi^2|\boldsymbol{b},\boldsymbol{A})$.

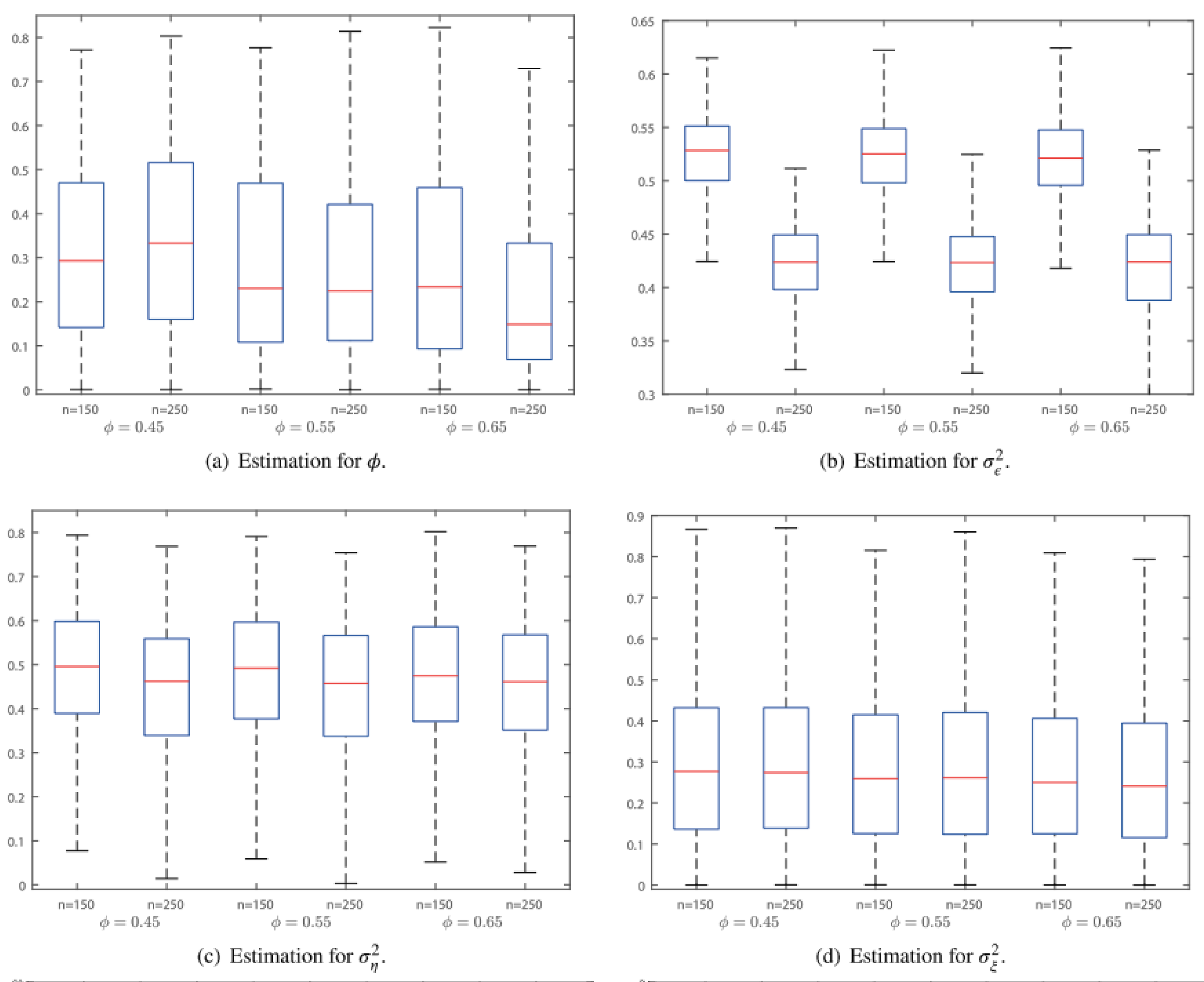
Random coefficient state-space model

$$
\begin{aligned}
y_t &= l_{t-1} + A_t b_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}\left(0,\sigma_\epsilon^2\right),\\
l_t &= l_{t-1} + A_t b_{t-1} + \eta_t, \eta_t \sim \mathcal{N}\left(0,\sigma_\eta^2\right),\\
b_t &= A_t b_{t-1} + \xi_t, \xi_t \sim \mathcal{N}\left(0,\sigma_\xi^2\right),
\end{aligned} \tag{1}
$$

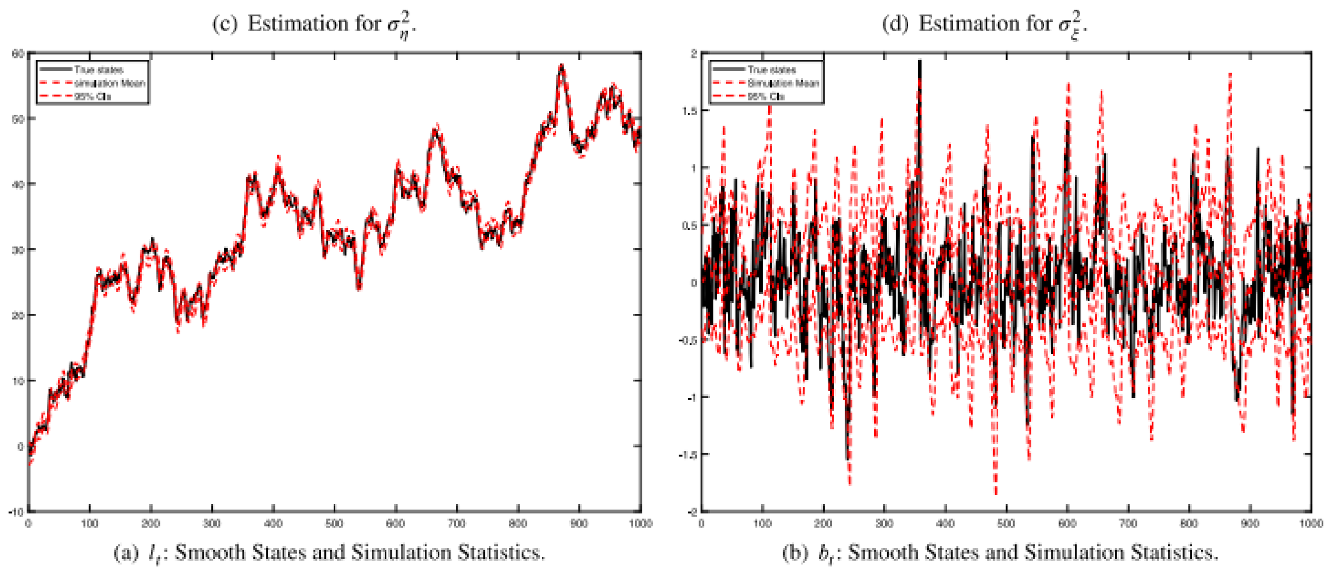
where $t = 1, \dots, T$, and $\mathcal{N}(\cdot,\cdot)$ denotes independent and identically normal distribution, y_t is the observation at time t, l_t is the stochastic trend, b_t is the slope of its stochastic trend, the term A_t is a sequence of independent, identically distributed binary random variables with probabilities:

$$P(A_t = 1) = \phi, P(A_t = 0) = 1 - \phi, 0 \le \phi \le 1.$$

Simulation study and application to credit



(a) Estimation for ϕ. (b) Estimation for σ_ϵ^2. (c) Estimation for σ_η^2. (d) Estimation for σ_ξ^2.

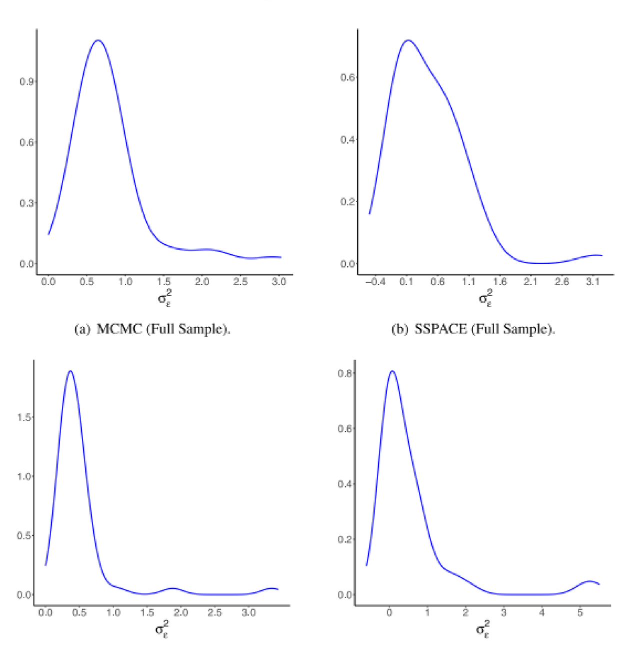


(a) l_t: Smooth States and Simulation Statistics. (b) b_t: Smooth States and Simulation Statistics.

Out-of-sample forecast comparison of RC-MCMC vs. RCSPACE based on quarterly BIS credit data.

	No. ahead step	Mean ratio	Median ratio	Ranking
Full sample	$h = 1$	0.983	1.092	0.5
	$h = 2$	0.899	0.916	0.625
	$h = 3$	0.890	0.837	0.708
	$h = 4$	0.868	0.714	0.708
	$h = 5$	0.936	0.999	0.604
	$h = 6$	1.002	1.066	0.479
	Mean	0.930	0.937	0.604
Short-term sample	$h = 1$	0.935	1.011	0.617
	$h = 2$	0.891	0.887	0.681
	$h = 3$	0.900	0.820	0.787
	$h = 4$	0.870	0.695	0.723
	$h = 5$	1.002	1.070	0.574
	$h = 6$	0.944	0.973	0.574
	Mean	0.924	0.909	0.660

Notes. In the short-term sample, the data set for Ireland has been removed as the trend in the data is not clear, 47 in total for short-term sample.



(a) MCMC (Full Sample). (b) SSPACE (Full Sample). (c) MCMC (Short-term Sample). (d) SSPACE (Short-term Sample).

- The RC-MCMC estimation method provides more precise parameter estimation.
- This method is not limited by optimization algorithms, leading to more realistic estimation results.
- The RC-MCMC method achieves more accurate forecast.
- It is particularly effective for data characterized by higher trend variability.

Conclusions

We introduce a **precision-based algorithm** to estimate parameters in the MSOE model, comparing its effectiveness with the RC-SSPACE method. Our analysis shows that RC-MCMC provides superior **accuracy** and **stability** in parameter estimation. Further empirical investigation using BIS data confirms that **RC-MCMC yields more realistic and reliable forecasting, particularly effective in handling models with frequent coefficient changes.**

Selected references

Chan, J., Grant, A.L., 2016. On the observed-data deviance information criterion for volatility modeling. J. Financ. Econom. 14 (4), 772–802pp.

Chan, J.C., Jeliazkov, I., 2009. Efficient simulation and integrated likelihood estimation in state space models. Int. J. Math. Model. Num. Optim 1 (1–2), 101–120.

Kim, S., Shephard, N., Chib, S., 1998. Stochastic volatility: Likelihood inference and comparison with arch models. Rev. Econom. Stud. 65 (3), 361–393.

Sbrana, G., Silvestrini, A., 2019. Random switching exponential smoothing: A new estimation approach. Int. J. Prod. Econ. 211, 211–220.

Sbrana, G., Silvestrini, A., 2020. Forecasting with the damped trend model using the structural approach. Int. J. Prod. Econ. 226, 107654.